Estimating Quarterly Fixed Asset Data

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Introduction

This document provides a detailed description for the construction of quarterly fixed asset data for corporate and noncorporate businesses, which is used as an input for Mejia (2021). Code and underlying data are available in an accompanying file.

Procedure

Fixed Asset data from the Bureau of Economic Analysis is published on an annual basis, which presents an issue for matching aggregate fixed asset data with firm-level data at a quarterly frequency. The key variables in the firm-level analysis are the net capital stock, gross investment, and depreciation. To obtain estimates of these at a quarterly frequency for the aggregate fixed asset data, I follow the methodology of McGrattan (2020). Because the law of motion of capital necessarily holds, i.e., $K_{t+1} = I_t + (1 - \delta_t)K_t$, it is required only to obtain estimates of two variables and the third follows. There is a categorical difference in the estimation procedure for a stock variable (K_t) versus a flow variable (I_t and $\delta_t K_t$). Whereas stock variables published at an annual frequency contain sufficient information about the timing of the value of the variable, flow variables do not. The value of the capital stock published at the end of year t gives the value of the capital stock at the end of Q4, but the value of a flow variable published at the end of period t contains no information about *when* the flows were initiated. In the extreme, it could be that all investment published by the BEA occurred in Q1 or Q4 and there is little reason to assume lack of seasonality.

To estimate either a stock or a flow, the first step is the same. Let Z_t be an annual variable, whether it is the capital stock, investment, or depreciation. Select X_t variables published from other sources available at quarterly frequency and used to make inferences about the quarterly value of Z_t, which I call $\hat{Z}_t$. The first step is to detrend all time series Z_t and X_t using the Hodrick-Prescott filter with smoothing parameter $\lambda = 1600$ for the quarterly series and $\lambda = 100$ for annual series. Then, to obtain quarterly estimates of $\hat{Z}_t$, we estimate A and

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B in the following state space system via maximum likelihood:

$$\begin{aligned} x_{t+1} &= Ax_t + B\epsilon_{t+1} \\ y_t &= Cx_t, \end{aligned} \tag{1}$$

where $x_t = \left[X_t, \hat{Z}_t, X_{t-1}, \hat{Z}_{t-2}, X_{t-2}, \hat{Z}_{t-3}, X_{t-3}, \hat{Z}_{t-1}, X_{t-4}, \hat{Z}_{t-4}\right]^T$, $y_t = \left[X_t, Z_t\right]^T$, and ϵ_t are normally distributed shocks. Coefficients are given by

$$A = \begin{bmatrix} a_1 & a_2 & \dots & a_j \\ I & 0 & \dots & 0 \\ 0 & I & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \dots & 0 \end{bmatrix}, \ B = \begin{bmatrix} b \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}, \ C = I \tag{2}$$

With estimates for $\left(\hat{A}, \hat{B}\right)$, it becomes possible to estimate forecasts $\hat{Z}_t = \mathbb{E}[Z_t | y_1, \dots, y_T]$ of annual data at a quarterly frequency by applying a Kalman smoother and then adding back the low-frequency Hodrick-Prescott trend to the estimated time series. At this point, the estimation procedure for the net capital stock is complete. To finish estimating flow variables, I add another step. Because I have estimated the flows as if they are stocks and therefore obtained estimates of depreciation and investment many magnitudes greater than would make sense given the value of the annual variable, I group estimated gross investment by year, divide each quarterly estimate by the sum of quarterly estimates, and multiply the result by the actual annual gross investment value. For example, suppose the published value of gross investment in 2010 is \$800B and I estimate that gross investment in 2010Q1 is \$150B, 2010Q2 is \$200B, 2010Q3 is \$250B, and 2010Q4 is \$200B. Then to estimate the flow value of gross investment undertaken in 2010Q1, I would take $\left(\frac{\$150B}{\$150B+\$220B+\$250B+\$200B}\right) \times \$800B = \$146.3B$, and so on for each year and set of four quarters. The estimate for depreciation then becomes mechanical:

$$\hat{D}_t = \hat{I}_t - \left(\hat{K}_t - \hat{K}_{t-1}\right), \tag{3}$$

where D is total depreciation in period t. Following this procedure produces estimates that are reasonable given the available data, particularly stocks. In Figure 1, I show the percent deviation between my procedure and what would happen if I employed a naive linear interpolation between annual figures for the total net capital stock of non-financial corporate institutions (Fixed Asset Table 4.7, Line 37). Compared to linear interpolation, my method captures much of the variability that would otherwise be missed.

I utilize the procedure outlined above to obtain quarterly estimates of Fixed Asset Tables 4.3, 4,6, and 4.7 Lines 37-44. The variables used to estimate series directly using the Kalman smoother method are given in Table 1. Table 4.7 Line 37 is the sum of 4.7 Lines 38-40 and Table 4.7 Line 41 is the sum of 4.7 Lines 42-44. Table 4.6 Lines 37-40 are estimated indirectly as in equation 3. Estimates for non-financial corporate entities begin in 1953 and estimates for non-financial non-corporate entities begin in 1960. Before getting impulse responses with the Mertens-Ravn proxy SVAR, I deflated each gross investment series using the GVA deflator as defined above.

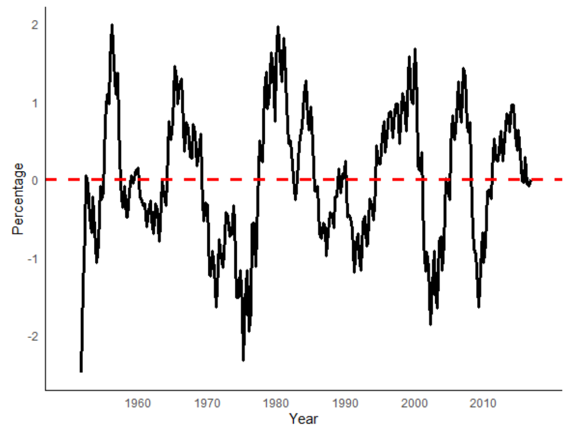

Figure 1: Percent deviation between my method and linear interpolation for estimation of Fixed Asset Table 4.7, Line 37.

Table	Line	Variables
4.3	37	TTAATASHCBSHNNCB, NCBWSPQ027S, NCBCFCQ027S (all FOF)
	38	ESATASHCBSNNCB, NCBWSPQ027S, NCBCFCQ027S (all FOF)
	39	HCVSNNWHCBSNNCB, NCBWSPQ027S, NCBCFCQ027S (all FOF)
	40	NCBNIPPHCB, NCBWSPQ027S, NCBCFCQ027S (all FOF)
4.7	38	NCBWSPQ027S (FOF), ESATASHCBSNNCB (FD, FOF), 1.14.19 (NIPA), 1.5.5.30 (NIPA)
	39	NCBWSPQ027S, HCVSNNWHCBSNNCB (FOF, FD), 1.14.19 (NIPA), 1.5.5.30 (NIPA)
	40	NCBWSPQ027S, NCBNIPPHCB (FOF, FD), 1.14.19 (NIPA), 1.5.5.37 (NIPA)
	42	NESABSNNB (FOF, FD), 1.5.5.30 (NIPA), NNBNVAQ027S (FOF)
	43	RCVSNWBSNNB (FOF, FD), 1.5.5.29 (NIPA), NNBNVAQ027S (FOF)
	44	NNBNIPPCCB (FOF, FD), 1.5.5.37 (NIPA), NNBNVAQ027S (FOF)

Table 1: Variables used to construct quarterly estimates of annual fixed asset series. Source is in parentheses. FOF refers to Flow of Funds from the Federal Reserve and NIPA refers to the National Income and Product Accounts published by the Bureau of Economic Analysis. FD means the variable is first-differenced.

References

McGrattan, E. R. (2020, 8). Intangible capital and measured productivity. *Review of Economic Dynamics 37*, S147–S166.

Mejia, J. (2021). Tax Shocks and Investment: Aggregate and Firm-Level Heterogeneity.